ERG2011A Tutorial 6 (Supp.)

Mathematica Output on Ploting Differential Equations

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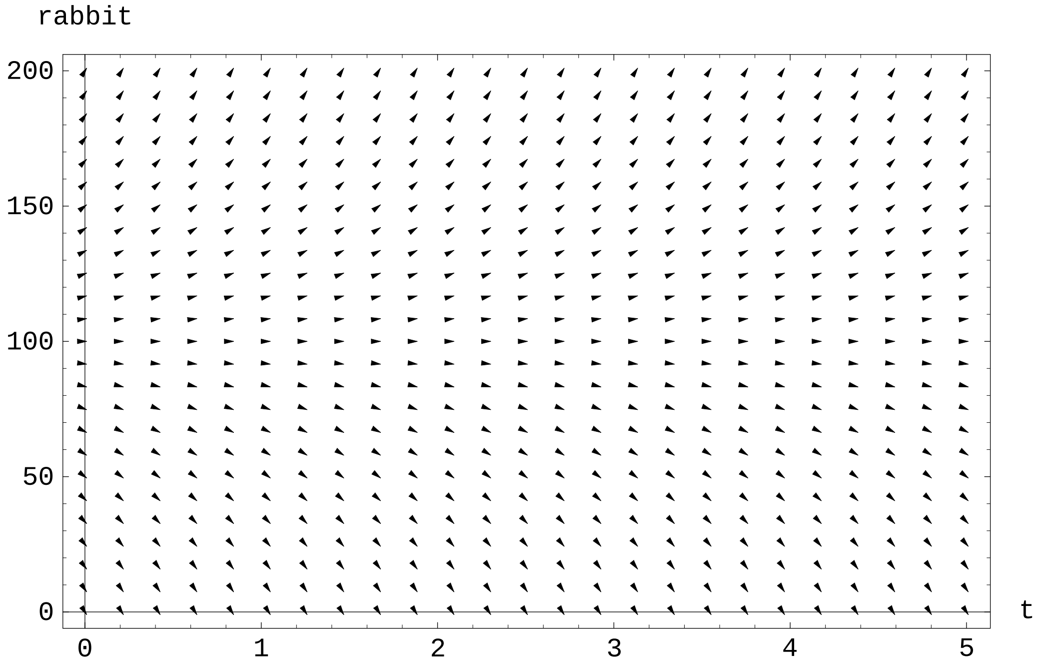
Direction Field

In[18]:=
```
<< Graphics`PlotField`
field = PlotVectorField[{1, rabbit − 100}, {t, 0, 5}, {rabbit, 0, 200},
  AspectRatio → 1 / GoldenRatio, Frame → True, HeadLength → 0.01, PlotPoints → 25,
  AxesLabel → {"t", "rabbit"}, Axes → True, ImageSize → 500, ScaleFactor → 1]
```



Out[19]= - Graphics -

In[3]:= `DSolve[rabbit'[t] == rabbit[t] − 100, rabbit[t], t]`

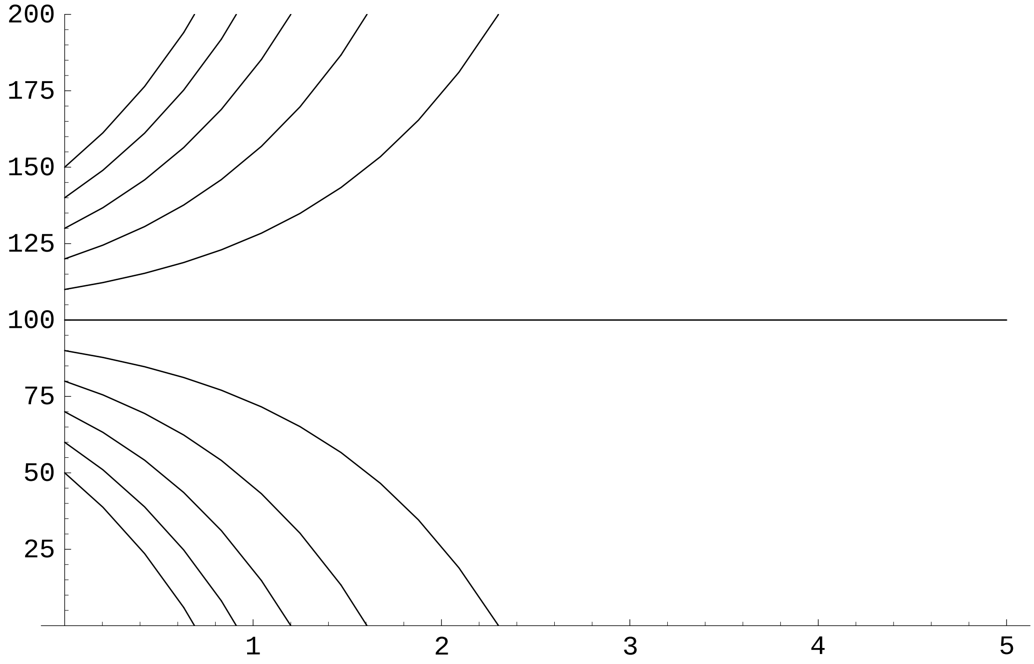
Out[3]= {{rabbit[t] → 100 + e^t C[1]}}

In[4]:= `rabbit[t_] = rabbit[t] /. First[First[%]]`

Out[4]= 100 + e^t C[1]

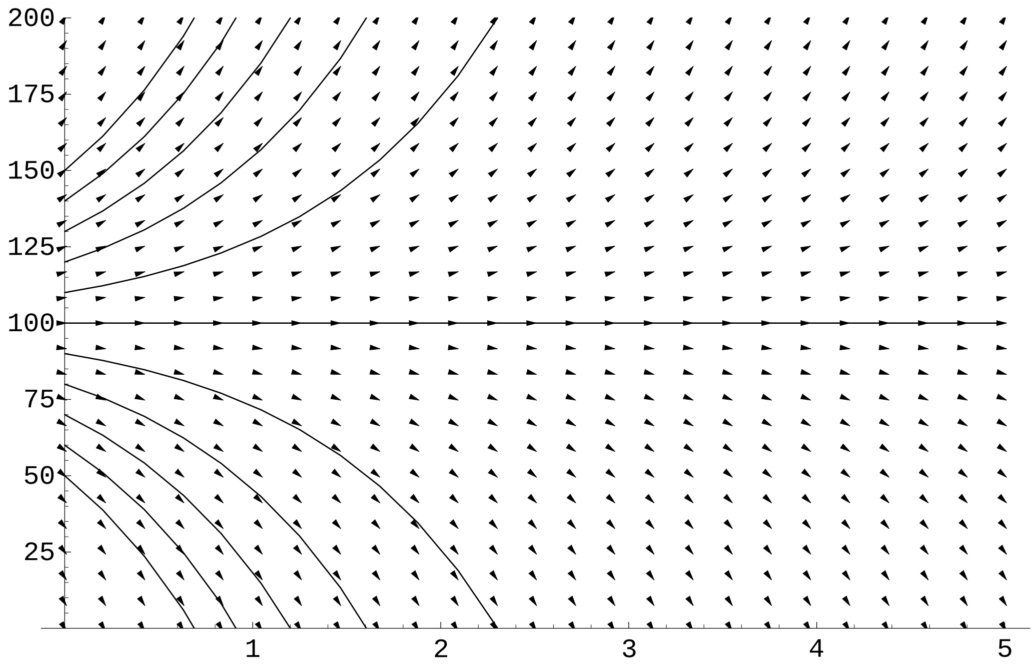
```
In[20]:= functions = rabbit[t] /. C[1] → Range[-50, 50, 10]
         lines = Plot[Evaluate[functions], {t, 0, 5}, PlotRange → {0, 200},
           AspectRatio → 1 / GoldenRatio, Axes → True, ImageSize → 500]
```

Out[20]= $\{100 - 50\, e^t,\ 100 - 40\, e^t,\ 100 - 30\, e^t,\ 100 - 20\, e^t,\ 100 - 10\, e^t,\ 100,\ 100 + 10\, e^t,\ 100 + 20\, e^t,\ 100 + 30\, e^t,\ 100 + 40\, e^t,\ 100 + 50\, e^t\}$



Out[21]= - Graphics -

```
In[22]:= Show[lines, field]
```



Out[22]= - Graphics -

Orthogonal Trajectories

Finding the constant c in terms of a particular point (x0,y0), and substitute back to the equation:

```
In[8]:=  eqn1 = y == c E^(x^2)
         particularpoint = Solve[eqn1, c] /. {x → x0, y → y0}
```

Out[8]= $y == c\, e^{x^2}$

Out[9]= $\{\{c \to e^{-x0^2}\, y0\}\}$

```
In[10]:=  eqn2 = First[eqn1 /. particularpoint]
```

Out[10]= $y == e^{x^2 - x0^2}\, y0$

Then, differentiate the whole equation and define m1=y'[x0], which is the slope at (x0,y0).
Afterwards, replace (x0,y0) with (x,y) to get the general solution of the derivative.

```
In[11]:=  de1 = D[eqn2 /. y → y[x], x] /. x → x0
          m1 = Last[de1] /. {x0 → x, y0 → y}
```

Out[11]= y′[x0] == 2 x0 y0

Out[12]= 2 x y

For the orthogonal trajectories, we find their slope m2 at point (x,y) by using the slope of the previous equation, m1

```
In[13]:=  m2 = -1 / m1 /. y → y[x]
```

Out[13]= $-\frac{1}{2\, x\, y[x]}$

Because m2 is the slope, we define y'[x]=m2 and solve for y[x].

```
In[14]:=  trajsol = DSolve[y'[x] == m2, y[x], x]
```

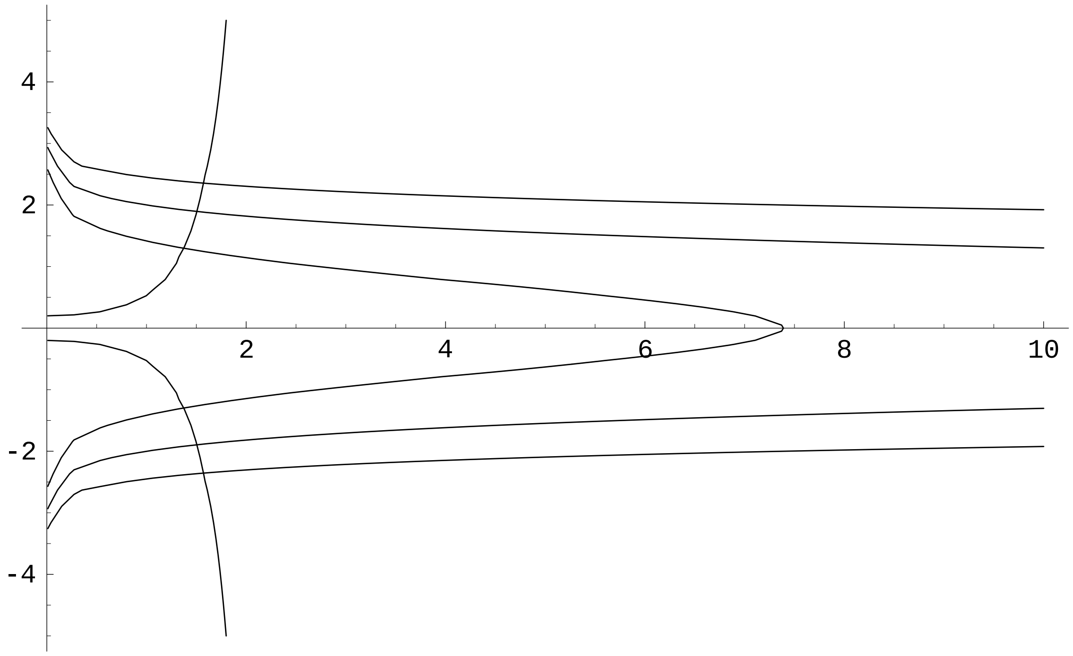
Out[14]= $\{\{y[x] \to -\sqrt{2\, C[1] - Log[x]}\}, \{y[x] \to \sqrt{2\, C[1] - Log[x]}\}\}$

```
In[15]:=  trajs = y^2 == Last[Last[Last[trajsol]]]^2
```

Out[15]= $y^2 == 2\, C[1] - Log[x]$

Now we get the equation in implicit form. So we plot it using ImplicitPlot.

```
In[31]:=  << Graphics`ImplicitPlot`
          ImplicitPlot[
           {trajs /. C[1] → 1, trajs /. C[1] → 2, trajs /. C[1] → 3, eqn1 /. c → -1 / 5, eqn1 /. c → 1 / 5},
           {x, 0.01, 10}, {y, -5, 5}, ImageSize → 500, AspectRatio → 1 / GoldenRatio]
```



Out[32]= - Graphics -