ERG2011A Tutorial 1: Fundamental Vector Calculus

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1 Key Information

Course homepage: http://course.ie.cuhk.edu.hk/~erg2011a/

Newsgroup: news://news.erg.cuhk.edu.hk/cuhk.erg.2011a

Lecturer: Prof Kim Chang-su (SHB 707)

Tutors: Kong-wa (SHB 803) and Adrian (SHB 826A)

Lectures: M3-4 (SC LG23), H3 (LHC 103)

Tutorials: M2, W5

2 Vectors as in High School

- Vectors: (   ) and (   )

  - Example: "North 30m"

- Vectors in 2D: in Cartesian plane

  - Unit vectors: **i** (   ) and **j** (   )
  - Handwritten form: $\vec{i}$ and $\vec{j}$
  - Ordered-pair form: (   ) $= (x, y)$

- Vector (   ) product / Inner product

  - Example of use in Physics: (   ), i.e. $W = Fs\cos\theta$
    * Vector notation: $W = \vec{F} \cdot \vec{s}$
  - Example: $\mathbf{v_1} = 4\mathbf{i} + 6\mathbf{j}$ and $\mathbf{v_2} = 3\mathbf{i} - 3\mathbf{j}$ then $\mathbf{v_1} \cdot \mathbf{v_2} =$

    or alternatively, $\mathbf{v_1} \cdot \mathbf{v_2} =$

  - Vector dot product can be generalized for n-dimensional cases, e.g. $n = 4$

  $$(u_1, u_2, u_3, u_4) \cdot (v_1, v_2, v_3, v_4) \quad = \quad u_1 v_1 + u_2 v_2 + u_3 v_3 + u_4 v_4$$

- When we do a dot product of a vector with itself, we get the square of its (   )
  * Denoted by: $\mathbf{x} \cdot \mathbf{x} =$
  * Sometimes, we call this value the Euclidean norm of vector $\mathbf{x}$, denoted by $||\mathbf{x}||$

- Vector cross product

  - Example of use in Physics: Force according to Faraday's Law: $F = BI\ell \sin\theta$

  - Vector cross product is defined only in (   ) space
  - Magnitude of vector cross product: $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$
  - Please note that, the θ here should be:

    Reason: To keep (   )

  - Vector triple product:
    $$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

3   Vector Functions and Scalar Functions

- Function = Mapping of an input to an (   )
- Notation: $f : \mathbb{R} \to \mathbb{R}$

- Classification of function (1) — Real number as input and output
  - Example: $f(x) = \sin(x)$
    * Input domain:

    * Output range:

  - Example: $f(x) = \sqrt{x}$
    * Input domain:

    * Output range:

- Classification of function (2) — Vector as input and output
  - Example: $\vec{a}(\vec{F}) = \vec{F}/m$
    * Input domain:

    * Output range:

- Example: $\vec{c}(\vec{x}, \vec{y}) = \vec{x} \times \vec{y}$
  * Input domain:

  * Output range:

- Classification of function (3) — Real number as input, vector as output
  - Example: $\vec{v}(t) = 4t\vec{i} + \frac{2}{t}\vec{j}$
    * Input domain:

    * Output range:

- Classification of function (4) — Vector as input, real number as output
  - Example: $f(\vec{x}) = |\vec{x}| = \sqrt{\vec{x} \cdot \vec{x}}$
    * Input domain:

    * Output range:

- Summary:

$$\text{Scalar function}: \quad \left\{ \begin{array}{c} f(x) = \sin(x) \\ f(x) = \sqrt{x} \\ f(\vec{x}) = |\vec{x}| = \sqrt{\vec{x} \cdot \vec{x}} \end{array} \right.$$

$$\text{Vector function}: \quad \left\{ \begin{array}{c} \vec{a}(\vec{F}) = \vec{F}/m \\ \vec{c}(\vec{x}, \vec{y}) = \vec{x} \times \vec{y} \\ \vec{v}(t) = 4t\vec{i} + \frac{2}{t}\vec{j} \end{array} \right.$$

  - We learnt about scalar function in high school
  - We are interested in vector functions in ERG2011A, especially functions in class (4)
  - We call the study of integration and differentiation of vector functions the **vector calculus**